One-gluon-exchange quark scattering and large-p_T inclusive data*

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Calculations based on the idea of asymptotic freedom enable one to estimate an effective quark-gluon coupling from experimental observations of scaling violations in deep-inelastic scattering, from Okubo-Zweig-Iizuka-rule-forbidden decays of the ψ and the ψ', and from corrections to the parton model for $\sigma(e^+e^-\rightarrow \text{hadrons})$. These estimates then make it possible to normalize the one-gluon-exchange Born term for quark-quark elastic scattering and, within the context of hard-scattering models for hadronic collisions, obtain a lower limit on large-p_T inclusive cross sections. The theoretical lower limit for the cross section is found to be slightly below CERN ISR data on $pp\rightarrow\pi^0X$ at $\sqrt{s}=53$ GeV, $p_T\geq 6$ GeV. Further measurements at the CERN ISR or at projected new accelerators may be sensitive to the presence of this mechanism. Such measurements will be extremely important in testing the underlying unity of diverse reactions within the general framework of quark-gluon dynamics. They may also provide the most direct determination of the effective hadronic coupling constant.

I. INTRODUCTION

The idea that a scale-invariant quark-quark scattering mechanism could be an important source of large-p_T hadrons was first discussed by Berman and Jacob.[1] Predictions have since been explicitly calculated by Berman, Bjorken, and Kogut[2] and by Ellis and Kislinger.[3] It was immediately noticed that this type of mechanism would lead to invariant cross sections displaying naive dimensional scaling,

$$E\,d\sigma/d^3p(pp\rightarrow\pi X)\sim\frac{f(x_T,\theta)}{p_T{}^4}\ , \tag{1.1}$$

where $x_T=2p_T/s^{1/2}$. Since available data[4] do not show this behavior, various alternate models[1-5] have been invented to describe large-p_T processes. We mention two of these many models as examples of how large-p_T hadronic phenomenology has been approached. One is the modified quark-scattering model (MQSM), advocated by Field and Feynman,[6] in which it is assumed that the fundamental hard process involves quark-quark elastic scattering ($qq\rightarrow qq$), but where the qq cross section is determined empirically rather than being given by theoretical consideration. Another approach, developed in a series of papers by Blankenbecler, Brodsky, and Gunion,[7] is the constituent-interchange model (CIM), which argues that current data on $pp\rightarrow\pi X$ can be understood from an underlying $qM\rightarrow qM$ process.

These models have enjoyed considerable phenomenological success, but the problem of the scaling term (1.1) remains. Where is it? Quantum chromodynamics (QCD) seems to predict the presence of a scaling term such as (1.1) (modified slightly by logarithmic corrections). The issue is important, since one of the attractive features of QCD is that it stands a chance of providing a *complete* theory of the strong interactions.[8] The ideas of asymptotic freedom and QCD have already supplied a theoretical underpinning for the parton model's success in deep-inelastic lepton scattering. It is natural to expect that QCD will provide a guide to other hard processes. In the case of large-p_T hadron collisions, a plausible suggestion is that current data from Fermilab and the CERN ISR are in an intermediate-p_T kinematic range and that we simply have not yet seen the true asymptotic behavior.

We can test this suggestion quite directly. The general expression for a hard-scattering parton model[5] is

$$\frac{E\,d\sigma}{d^3p}\,(pp\rightarrow\pi X)\sim\frac{1}{\pi}\sum_{ab\rightarrow cd}\int dx_a dx_b G_{a/p}(x_a)G_{b/p}(x_b)\,\frac{dz}{z^2}\,D_{\pi/c}(z)\left[\hat{s}\,\frac{d\sigma}{d\hat{t}}\,(ab\rightarrow cd)\right]\delta(\hat{s}+\hat{t}+\hat{u})\ , \tag{1.2}$$

where $\hat{s}=x_a x_b s$, $\hat{t}=x_a t/z$, and $\hat{u}=x_b u/z$. In (1.2), $G_{a/p}(x_a)$ is the probability of finding the constituent a carrying a fraction x_a of the incident proton's momentum, and $D_{\pi/c}(z)$ is the "decay distribution" for finding a pion with a fraction z of the constituent c's momentum.

We believe that the contribution to (1.2) from the one-gluon-exchange (OGE) "Born term" can be normalized using information obtained from other sources. The functions $G_{q/p}(x)$ and $D_{\pi/q}(z)$

are determined from experimental data on lepton processes in the manner described by Field and Feynman in Ref. 6. We will restrict our attention to the process $pp\to\pi X$, where the uncertainties in G and D are comparatively small. The only component of the model remaining to be determined is the quark-quark scattering cross section $d\sigma/d\hat{t}$. After summing over color the contribution to qq scattering from the exchange of a vector gluon is

$$\frac{d\sigma}{d\hat{t}}(q_a q_b \to q_a q_b) \sim \frac{2}{9}\,\frac{2\pi\alpha_s^2}{\hat{s}^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\delta_{ab}\left(\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}-\frac{2}{3}\frac{\hat{s}^2}{\hat{t}\hat{u}}\right)\right], \quad (1.3)$$

where a and b label the flavors of the quarks. The u-channel graph is only present when the initial quarks are identical. We also include the s-channel graphs for the $q\bar{q}$ initial state,

$$\frac{d\sigma}{d\hat{t}}(q_a\bar{q}_a \to q_b\bar{q}_b) \sim \frac{2}{9}\,\frac{2\pi\alpha_s^2}{\hat{s}^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}+\delta_{ab}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}-\frac{2}{3}\frac{\hat{u}^2}{\hat{s}\hat{t}}\right)\right]. \quad (1.4)$$

These s-channel terms are very small for $pp\to\pi X$.

In the spirit of asymptotic freedom we replace α_s by an effective coupling $\alpha_s(-\hat{t})$ which vanishes as $-\hat{t}\to\infty$. As we will discuss in the next section, we can achieve a fair estimate of $\alpha_s(-\hat{t})$ within the framework of asymptotic freedom in a variety of ways. Once this is done, the OGE term is completely normalized.

Accepting the basic assumptions of the hard-scattering model concerning the neglect of interference effects, we believe this OGE term should be considered a *lower limit* for the inclusive cross section. We can therefore test the consistency of the whole approach by checking that the large-p_T inclusive data from the CERN ISR (which behave approximately like p_T^{-8}) are above the limit given by (1.2)–(1.4), and the best estimate of the strong-coupling constant. Moreover, we can try to predict the values of p_T and s at which the OGE term, which falls like a smaller power of p_T^{-1}, should begin to dominate the inclusive cross section.

We describe in this article a calculation of the inclusive cross section for $pp\to\pi^0 X$ using (1.2)–(1.4). In Sec. II we discuss some of the ways in which people have estimated the effective quark-gluon coupling α_s, and we select a range of values for $\alpha_s(-\hat{t})$ to use in (1.3). In Sec. III we evaluate (1.2) using the parametrizations of $G_{q/p}(x)$ and $D_{\pi/q}(z)$ obtained by Field and Feynman[6] and we compare the results to data from the CERN ISR. We also parametrize the scaling violations predicted in QCD by modifying the distribution functions, $G_{q/p}(x)\to G_{q/p}(x,-\hat{t})$, in a manner consistent with the Q^2 dependence of deep-inelastic lepton scattering. The inclusive data at $\sqrt{s}=53$ GeV and $p_T\geq 6$ GeV are roughly the same order of magnitude as the OGE term calculated in this way. This simple exercise suggests that data on large-p_T processes at higher p_T and/or s can be used to determine the effective strong coupling α_s. We show how data at large p_T from proposed new accelerators should be sensitive to the presence of OGE. Section IV discusses some of the ambiguities in our application of asymptotic-freedom ideas to large-p_T inclusive scattering and presents our conclusions.

II. ESTIMATES OF THE QUARK-GLUON COUPLING

In order to test our idea that the QCD one-gluon-exchange amplitude should provide a lower bound for large-p_T inclusive data, we need to know the approximate size of the strong-coupling constant. In the deep Euclidean region (where all momenta are large and spacelike), one can use renormalization-group techniques to see that the effective coupling constant in the standard $SU(3)_{\text{flavor}}\times SU(3)_{\text{color}}$ version of QCD satisfies the equation[9]

$$\frac{d\alpha_s(Q^2)}{d(\ln Q^2)}=-\frac{27}{12\pi}\,\alpha_s^2(Q^2)+O(\alpha_s^3)\,, \quad (2.1)$$

which has the solution

$$\alpha_s(Q^2)=\frac{\alpha_s(M_0^2)}{1+(27/12\pi)\alpha_s(M_0^2)\ln(Q^2/M_0^2)}\,, \quad (2.2)$$

provided that

$$\alpha_s(M_0^2)\ll 1,\quad Q^2\geq M_0^2\,. \quad (2.3)$$

This form for $\alpha_s(Q^2)$ is based on the approximations that $SU(3)_{\text{flavor}}$ is an exact symmetry and that the three flavors of quarks are all massless. The approximations are not absolutely necessary; renormalization-group techniques can be used to incorporate thresholds for the excitation of heavy quarks into an expression[10] analogous to (2.2). However, the highest Q^2 for which we will use (2.2) will be approximately 100–200 GeV^2, and in this regime the corrections to (2.2) due to the excitation of charmed quarks are still only 1–2%. At low values of Q^2 (5–10 GeV^2) the change of α_s due to the threshold for strange quarks is similarly small. We will henceforth ignore these small corrections and use (2.2) as a guide to the Q^2 dependence of α_s.

The fact that we will apply (2.2) in kinematic regions which are far from the deep Euclidean regime is another possible source of concern. In particular, the qq scattering in large-p_T hadronic processes can occur with quarks near their mass shells, whereas particles in the deep Euclidean region are far off the mass shell. Moreover, several of the estimates of α_s which we will review shortly are performed with the quark momenta timelike instead of spacelike. The renormalization-group approach, however, suggests that the effective coupling is asymptotically a function only of the magnitude of Q^2,

$$\alpha_s(Q^2) \cong \alpha_s(|Q|^2) \ . \tag{2.4}$$

We will assume throughout that (2.4) is valid to order α_s^2.

There is one undetermined parameter in (2.2), $\alpha_s(M_0^2)$, the value of the coupling at an arbitrary normalization point. There are several methods within the framework of asymptotic freedom to determine $\alpha_s(M_0^2)$. We will discuss three of these approaches.

A. OZI-rule-forbidden decays

One of the most frequently quoted estimates of the effective strong coupling involves the use of quark-gluon QCD and asymptotic freedom to explain the decays of the $\psi(3.1)$ and the $\psi'(3.7)$ which violate the Okubo-Zweig-Iizuka (OZI) rule.[11] The decay width for a charmed-quark-antiquark pair to annihilate through a three-gluon state into ordinary hadrons is[12,13]

$$\Gamma^{\rm OZI}_{\psi\to h} \cong |\psi(0)|^2 \frac{16}{9\pi}(\pi^2-9)\frac{5}{18}\frac{4\alpha_s^3}{M_\psi^2} \ , \tag{2.5}$$

where $\psi(0)$ is the nonrelativistic quark wave function and M_ψ is the mass of the system which is decaying.[14] The quantity $\Gamma^{\rm OZI}_{\psi\to h}$ can be extracted directly from the data,

$$\Gamma^{\rm OZI}_{\psi\to h} = \Gamma^{\rm tot}_\psi - (R+2)\Gamma_{\psi\to e^+e^-} \ . \tag{2.6}$$

We can eliminate the dependence of (2.5) on $|\psi(0)|^2$ by observing that the leptonic width of the ψ through one photon is[12,13,15]

$$\Gamma_{\psi\to e^+e^-} = |\psi(0)|^2\, 16\left(\frac{2\alpha}{3}\right)^2 \frac{1}{M_\psi^2} \ , \tag{2.7}$$

where $\alpha \cong \frac{1}{137}$. Taking $R(M_\psi)=2.5$, $\Gamma^{\rm tot}_\psi = 69$ keV, and $\Gamma_{\psi\to e^+e^-} = 5.0$ keV,[16] we find

$$\alpha_s(M_\psi^2) = \left[\frac{\Gamma^{\rm OZI}_{\psi\to h}}{\Gamma_{\psi\to e^+e^-}}\frac{18\pi\alpha^2}{5(\pi^2-9)}\right]^{1/3} = 0.20 \pm 0.02 \ , \tag{2.8}$$

where the error reflects only the uncertainty of the data. Motivated by Kinoshita's theorem,[17] we guess that this analysis has estimated $\alpha(Q^2)$ at $Q^2 = M_\psi^2$.

We can also approach the OZI-rule-violating decays of the ψ' in the same spirit. However, it is not easy to extract $\Gamma^{\rm OZI}_{\psi'\to h}$ from the data because there are many competing channels involving cascades through other charmonium states (e.g., $\psi' \to \psi\pi^+\pi^- \to$ hadrons). Nevertheless, it is possible to examine specific channels for noncascading ψ' decays, $\psi' \to \{h\}$. From the preceding analysis,

$$\frac{\Gamma_{\psi'\to\{h\}}}{\Gamma_{\psi'\to e^+e^-}} \cong \left[\frac{\alpha_s(M_{\psi'}^2)}{\alpha_s(M_\psi^2)}\right]^3 \frac{\Gamma_{\psi\to\{h\}}}{\Gamma_{\psi\to e^+e^-}} \ , \tag{2.9}$$

and from the experimental ψ and ψ' decay rates[16] for $\{h\} = \{2\pi^+2\pi^-\pi^0\}$, $\{K^+K^-\pi^+\pi^-\}$, and $\{p\bar{p}\}$, we find

$$\alpha_s(M_{\psi'}^2) = 0.20 \pm 0.04 \ . \tag{2.10}$$

Although this quark-gluon approach is an attractive model for OZI-rule violations, it is not clear that we understand the ideas behind asymptotic freedom well enough to apply this technique to a complicated process involving three gluons. A clue that we may not completely understand OZI-rule violations in this manner can be obtained from the prediction[18]

$$\frac{\Gamma_{\eta_c\to h}}{\Gamma_{\psi\to h}} = \frac{27\pi}{5(\pi^2-9)\alpha_s} \cong \frac{19.5}{\alpha_s} \ . \tag{2.11}$$

If we identify the η_c with the state observed at 2.8 GeV, then this relation may be in conflict with its inferred width.[19] We should also be worried that the mass splittings and the γ-transition rates predicted in charmonium schemes incorporating this value of α_s are in some conflict with experiment.[19]

B. $\sigma(e^+e^- \to \text{hadrons})$

Another method of determining α_s involves the approach to scaling of e^+e^- scattering. If we normalize $\sigma(e^+e^- \to \text{hadrons})$ to the QED cross section $\sigma(e^+e^- \to \mu^+\mu^-)$ in the usual way, we can take into account the leading-order corrections to QCD and write[18]

$$\begin{aligned} R(s) &= \frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma_{\rm QED}(e^+e^-\to\mu^+\mu^-)} \\ &\cong \tfrac{3}{2}\sum \lambda_a^2\beta_a(3-\beta_a^2)[1+\tfrac{4}{3}\alpha_s(s)f(\beta_a)+\cdots] \\ &\quad + \tfrac{1}{2}\sum_{\substack{\text{heavy}\\ \text{leptons}}} \beta_l(3-\beta_l^2) \ , \end{aligned} \tag{2.12}$$

where λ_a is the quark charge, β_a is the quark velocity, $\beta_a = (1-4m_a^2/s)^{1/2}$, and $f(\beta)$ is Schwinger's function[20]

$$f(\beta) \cong \frac{\pi}{2\beta} - \left(\frac{3+\beta}{4}\right)\left(\frac{\pi}{2}-\frac{3}{4\pi}\right) \ . \tag{2.13}$$

The published value[21] of R from SPEAR is $R=5.3 \pm 0.5$ at $\sqrt{s} = 6$ GeV. Taking the charmed-quark mass to be in the range $m_c = 1.5$–1.8 GeV, and including one heavy lepton of mass 1.8 GeV, we find

$$\alpha_s(36\ \mathrm{GeV}^2) = 0.6 \pm 0.4\ . \tag{2.14}$$

The large uncertainty of this estimate for α_s is primarily due to the (systematic) experimental uncertainty of R. Preliminary results from DESY[22] indicate that R may be 15% lower than the values measured at SPEAR; this change in R would reduce the estimate (2.14) to $\alpha_s \cong 0.2$.

Shankar[23] has also analyzed the corrections to $R(s)$ and has extracted values of α_s which are consistent with (2.14). He uses experimental data to determine the integral

$$\Omega(s) = \int_{4m_\pi^2}^{s} R(s')ds'\ , \tag{2.15}$$

and then he invokes (approximate) analyticity arguments to equate (2.15) to another (contour) integral. We do not know exactly how to evaluate the uncertainties of this procedure, but we are suspicious of claims that this strategy leads to a more accurate value of α_s than the simple approach which we have used in (2.12).

C. Scaling violations

One of the most characteristic predictions of an asymptotically free theory is the pattern of scaling violations predicted for deep-inelastic leptoproduction. The form of these scaling violations depends on the magnitude of α_s, and so analyses of leptoproduction data should enable us to obtain an estimate of α_s. One of the most thorough studies of electroproduction data within the framework of QCD has been performed by De Rújula, Georgi, and Politzer.[24] They analyze data[25] in the range $1 < Q^2 < 16\ \mathrm{GeV}^2$ and rely on the Nachtmann variable[26]

$$\xi = \frac{2x}{1 + (1 + 4x^2m^2/Q^2)^{1/2}}\ , \tag{2.16}$$

where $x = Q^2/2m\nu$ is the Bjorken variable, to describe the corrections to scaling expected in the small-α_s limit. Rewriting (2.2) as

$$\alpha_s(Q^2) = \frac{12\pi}{27\ln(Q^2/\Lambda^2)}\ , \tag{2.17}$$

they find that $\Lambda = 0.5$ GeV gives a good fit to νW_2 of the proton for $x \gtrsim 0.33$. Selecting a value of Q^2 to represent the range of the data they analyze, we find

$$\alpha_s(8\ \mathrm{GeV}^2) = 0.40 \pm 0.16\ , \tag{2.18}$$

where we have used α_s^2 as a crude estimate of uncertainty. An indication that all may not be well in this analysis is that their predictions for $\sigma_L(Q^2, \omega)/\sigma_T(Q^2, \omega)$ are systematically lower than experimental measurements.

In an independent analysis of the violations of Bjorken scaling, Johnson and Tung[26] concentrate on the quantity $\partial \ln\nu W_2(x, Q^2)/\partial \ln Q^2$ in the small-x region of muon-hadron scattering.[27] The data in this regime are not as good as in those studied by De Rújula, Georgi, and Politzer, but there does seem to be a clear nonscaling signal. Including the possibility of new-particle production in their analysis, Johnson and Tung obtain

$$\alpha_s \cong \frac{0.21}{1 + 0.14\ln Q^2} \tag{2.19}$$

as an upper limit for α_s. Since their analysis involves extrapolating the data to $Q^2 = 10\ \mathrm{GeV}^2$, we choose this point to evaluate their result, and find

$$\alpha_s(10\ \mathrm{GeV}^2) = 0.16 \pm 0.03\ . \tag{2.20}$$

Again, we have used α_s^2 as an estimate for the uncertainty.

An analysis by Parisi[28] also discusses scaling violations and gives a range

$$\alpha_s(6\ \mathrm{GeV}^2) \cong 0.25-0.50\ . \tag{2.21}$$

The phenomenological difficulties involved in extracting α_s from leptoproduction data are considerable. The important problems involve fitting data from different experiments, inverting moment relations for the structure functions, distinguishing between the low-energy *approach* to scaling and the asymptotic *corrections* to scaling, and estimating charm production. While all these effects make it difficult to be precise, it is significant that the overall pattern of scaling violations supports the predictions of asymptotic freedom. It is also significant that the range of values obtained for $\alpha_s(Q^2)$ in analyses of scaling violations are roughly consistent with those obtained from OZI-rule violations and ($e^+e^- \to$ hadrons).

D. Parametrization of $\alpha_s(Q^2)$

The estimates for $\alpha_s(Q^2)$ discussed above are shown in Fig. 1 as data points. We now choose a *range* of $\alpha_s(Q^2)$,

$$\begin{aligned} \alpha_s^{\max}(Q^2) &= \frac{0.50}{1 + 0.36\ln(Q^2/4)}\ , \\ \alpha_s^{\min}(Q^2) &= \tfrac{1}{2}\,\alpha_s^{\max}(Q^2)\ , \end{aligned} \tag{2.22}$$

which we feel is compatible[29] with the data, and we show this range in Fig. 1 as dashed lines. The upper limit for $\alpha_s(Q^2)$ agrees with the estimate of De Rújula, Georgi, and Politzer, and the lower limit with the estimate of Johnson and Tung and with the OZI estimates from ψ decay.

Also displayed in Fig. 1 as a solid line is the "effective coupling" obtained by comparing the OGE expression (1.3) for $d\sigma/d\hat{t}$ with the empirical

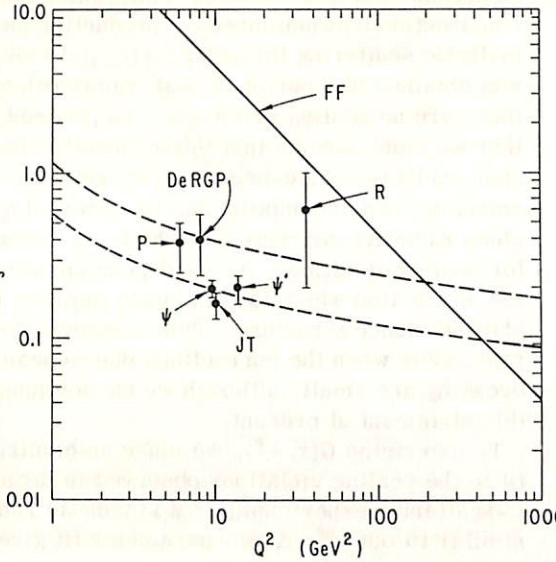

FIG. 1. The effective quark-gluon coupling $\alpha_s(Q^2)$. The "data" points are from analyses of the following processes: OZI-rule-forbidden decays (ψ, ψ'), Eqs. (2.8) and (2.10); corrections to the parton model for $\sigma(e^+e^- \to \text{hadrons})$ (R), Eq. (2.14); and scaling violations in deep-inelastic scattering [DeRGP, Eq. (2.18), JT, Eq. (2.20), and P, Eq. (2.21)]. The dashed lines delimit the range of $\alpha_s(Q^2)$ which we feel is consistent with these analyses. Shown as a solid line (FF) is the quark-gluon coupling which would yield a one-gluon-exchange term the same size as the MQSM of Field and Feynman (Ref. 6).

cross section of Field and Feynman.[16] The two expressions for $d\sigma/d\hat{t}$ are equated at fixed angle ($\hat{t}=\hat{u}=-\hat{s}/2$), with the result

$$\alpha_s^{\mathrm{FF}}(-\hat{t}) = \frac{20.6\ \mathrm{GeV}^2}{-\hat{t}} \quad . \tag{2.23}$$

There is a crossover between the FF and the OGE curves at $-\hat{t} = 200-600\ \mathrm{GeV}^2$. Using the kinematic approximation $-\hat{t} \cong 2p_T^{\,2}$, we expect that the OGE contribution to inclusive spectra will dominate for $p_T \gtrsim 10-15$ GeV. We will show this by explicit calculation in the next section.

III. HARD-SCATTERING PREDICTIONS FOR $pp \to \pi^0 X$

We can now evaluate the parton-model expression (1.2) for the inclusive production of pions. For simplicity we will first assume that the quark distributions $G_{q/p}(x)$ and the quark decay functions $D_{\pi/q}(z)$ scale exactly and are those given by Field and Feynman in Ref. 6. We will also examine the sensitivity of the calculation to small changes in these distributions. We will then relax the assumption of scaling for the distribution functions. This is consistent with the spirit of asymptotic freedom where $G_{q/p}(x, -\hat{t})$ is predicted to have $\hat{t}$ dependence, and with the observed scaling violations in leptoproduction.

A. The scaling limit

We want to examine two contributions to the hard-scattering-model expression for the inclusive cross section. The first is the one-gluon-exchange (OGE) term with $d\sigma/d\hat{t}$ given by (1.3) and (1.4) and $\alpha_s(-\hat{t})$ by (2.22). Of course, for the u-channel and s-channel terms we use $\alpha_s(-\hat{u})$ and $\alpha_s(\hat{s})$. As we are interested primarily in the region $p_T = 2-12$ GeV, at $\theta = 90°$ we are sensitive to $-\hat{t} \gtrsim 2p_T^{\,2}$, or a range in $-\hat{t}$ of 8–300 GeV^2. In this range of $\hat{t}$ we believe (2.22) is a good representation of the QCD estimates for α_s. The second contribution to the inclusive cross section which we will calculate is the MQSM of Field and Feynman (FF),[16]

$$\frac{d\sigma^{\mathrm{FF}}}{d\hat{t}} = \frac{2.3\times 10^6\ \mu\mathrm{b\,GeV}^6}{\hat{s}(-\hat{t})^3} \quad . \tag{3.1}$$

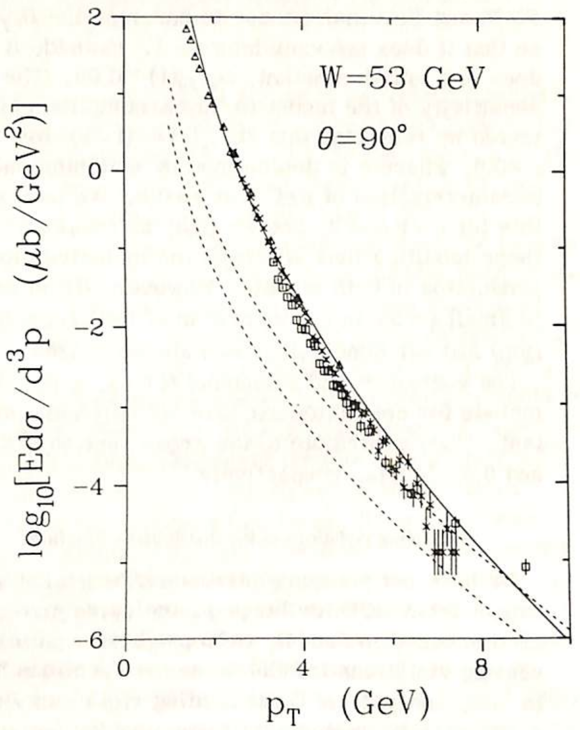

FIG. 2. Inclusive cross section for pion production, $E\,d\sigma/d^3p\,(pp \to \pi X)$, versus p_T at $\sqrt{s} = 53$ GeV, $\theta = 90°$. The data are $pp \to \pi^0 + X$ from Ref. 30 (triangles) and Ref. 31 (squares), and $pp \to (\pi^+ + \pi^-)/2 + X$ from Ref. 31 (crosses). The solid curve is calculated from the MQSM of Field and Feynman (Ref. 6). The dashed curves are the results of one-gluon-exchange corresponding to the range of $\alpha_s(-\hat{t})$ shown in Fig. 1 as dashed curves. Both models are calculated using the Field and Feynman quark distribution functions.

This model is known to give a good representation of data on many different large-p_T reactions in the Fermilab and ISR energy regime $\sqrt{s}=20-60$ GeV.

Figure 2 compares MQSM and our range for OGE to data[30,31] from the CERN ISR at $\sqrt{s}=53$ GeV and c.m. angle $\theta=90°$. Both models predict that π^0 yield is the average of π^+ and π^-, and so data on both π^0 and $(\pi^+ + \pi^-)/2$ are displayed. Since we have used the Field and Feynman fits to $G_{q/p}(x)$ and $D_{\pi/q}(z)$, the solid curve (MQSM) is merely a reproduction of a calculation reported in Ref. 6. This curve is an excellent parametrization of the data. The dashed curves show the range of the OGE contribution to π production corresponding to the range of α_s shown in Fig. 1 as dashed curves. The OGE term is surprisingly large, and there is a crossover between the predictions of the MQSM and OGE somewhere in the range $p_T=9$ to 15 GeV/c.

We have checked that the basic features of Fig. 2 are not sensitive to small changes in the distribution functions since, as discussed in Ref. 6, there are ranges of x and z where these functions are poorly determined by the data. In particular, Field and Feynman choose to parametrize $D_{\pi/q}(z)$ so that it does not vanish as $z\to 1$. Instead, it does to a small constant, $D_{\pi^0/u}(1)\cong 0.05$. The sensitivity of the model to this assumption can be tested by requiring that $D_{\pi^0/u}(z)=c(1-z)^n$ for $z>0.8$, where c is determined by matching onto the parametrization of Ref. 6 at $z=0.8$. We have done this for $n=1$ and 2, and we find, as expected, that these modifications decrease the inclusive pion production in both models. However, the effect is small ($\le 5\%$ in the logarithm of the cross section) and our conclusions remain unchanged.

The u-channel and s-channel terms, which we include for completeness, are actually unimportant. They contribute to the cross section at 15% and 0.5% levels, respectively.

B. Scaling violations in the distribution functions

We have not yet completed describing what we expect from QCD for large-p_T inclusive production. As discussed in Sec. II, QCD predicts a pattern of scaling violations for the structure functions.[9,18] In fact, analyses of these scaling violations in deep-inelastic lepton scattering provide one of the ways our estimate, (2.22), of $\alpha_s(-\hat{t})$ is determined. To be consistent, we should therefore take these scaling violations into account when calculating with (1.2) and replace

$$G_{q/p}(x)\to G_{q/p}(x,-\hat{t}) \,,$$
$$D_{\pi/q}(z)\to D_{\pi/c}(z,-\hat{t}) \,. \tag{3.2}$$

The replacement (3.2) involves an additional assumption. The derivation of (1.2), which specifies a connection between large-p_T production and deep-inelastic scattering through $\sum_q xG_{q/p}(x)=\nu W_2(x)$, was obtained in a parton-model framework where there are no scaling violations. To proceed further we must assume that this connection is unchanged by the scale-breaking corrections. This amounts to an assumption that the internal quark-gluon radiative corrections, which are responsible for scaling violations, do not depend sensitively on the interaction which is being used to probe the short-distance structure. This assumption seems reasonable when the corrections due to scale breaking are small, although we cannot quantify this statement at present.

To determine $G(x,-\hat{t})$, we make an empirical fit to the scaling violations observed in large-Q^2 μ-scattering experiments in a kinematic range similar to ours.[32] A two-parameter fit gives

$$\frac{\partial \ln G(x,Q^2)}{\partial \ln Q^2}\cong(0.2-1.0x) \,. \tag{3.3}$$

This form is very close to that obtained elsewhere in a fit to scale breaking in νN, $\bar{\nu}N$, ep, and μp interactions.[33] We will interpret this equation as giving an approximation for $G_{q/p}(x,-\hat{t})$ which is valid for each constituent of the proton taken separately. We neglect scaling violations in $D_{\pi/q}(z)$, so the substitution (3.2) can be written

$$G_{q/p}(x,-\hat{t})\cong G^{\rm FF}_{q/p}(x)\exp[(0.2-1.0x)\ln(-\hat{t})] \,,$$
$$D_{\pi/q}(z,-\hat{t})=D^{\rm FF}_{\pi/q}(z) \,, \tag{3.4}$$

where $G^{\rm FF}_{q/p}(x)$ and $D^{\rm FF}_{\pi/p}(z)$ are the parametrizations given in Ref. 6. The effect of (3.4) is then to enhance the low-p_T portion of the cross section, which is sensitive to low-x values in the integrations of Eq. (1.2) and to decrease the high-p_T range, which is sensitive to large-x values. For u-channel and s-channel terms we use $G_{q/p}(x,-\hat{u})$ and $G_{q/p}(x,\hat{s})$, and for interference terms we use the appropriate geometric means. These corrections to the small u-channel and s-channel terms are thoroughly insignificant.

The results of this calculation at $\sqrt{s}=53$ GeV are shown in Fig. 3. It is interesting that the scaling violations do not change drastically the quality of the Field and Feynman fit to the data. However, they do lower the predictions of the OGE calculation. The calculation of the OGE contribution with scaling violations (3.4) and α_s given by (2.22) now represents an improved estimate of a lower limit to the data. The data are clearly well above this limit.

We are obviously a long way from having evidence in the data for OGE, and so there is some point in discussing how one might obtain such evi-

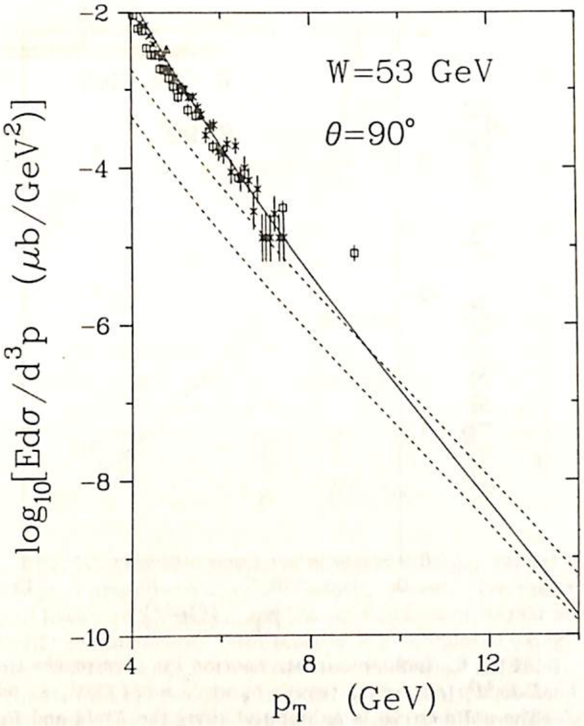

FIG. 3. Inclusive cross section for pion production, $E\,d\sigma/d^3p\,(pp \to \pi X)$, versus p_T at $\sqrt{s} = 53$ GeV, $\theta = 90°$. The data are the same as in Fig. 2. The solid curve is calculated using the Field and Feynman parametrization of $d\sigma/d\hat{t}$. The dashed curves are the results of one-gluon exchange corresponding to the range of $\alpha_s(-\hat{t})$ shown in Fig. 1 as dashed curves. Both models are calculated using scale-violating quark distribution functions, Eq. (3.4).

dence. One strategy might be to look for the p_T^{-4} behavior of (1.1). Actually, this power-law behavior is modified somewhat by logarithmic terms in $\alpha_s(-\hat{t})$ and $G_{q/p}(x, -\hat{t})$. Moreover, the most accessible quantity experimentally is the effective power of p_T^{-1} at *fixed s*,

$$N_{\text{eff}} = -\left.\frac{\partial \ln(E\,d^3\sigma/d^3p)}{\partial \ln p_T}\right|_s , \tag{3.5}$$

rather than at fixed $x_T = 2p_T/\sqrt{s}$. To illustrate this effect, if the inclusive cross section can be approximated by

$$E\frac{d\sigma}{d^3p} \cong A\frac{(1-x_T)^b}{p_T^{\,n}} , \tag{3.6}$$

then

$$N_{\text{eff}} \cong n + \frac{b x_T}{1-x_T} . \tag{3.7}$$

In Fig. 4 we plot N_{eff} [as defined in (3.5)] for MQSM and for OGE with nonscaling structure functions. We also show as dashed lines the results

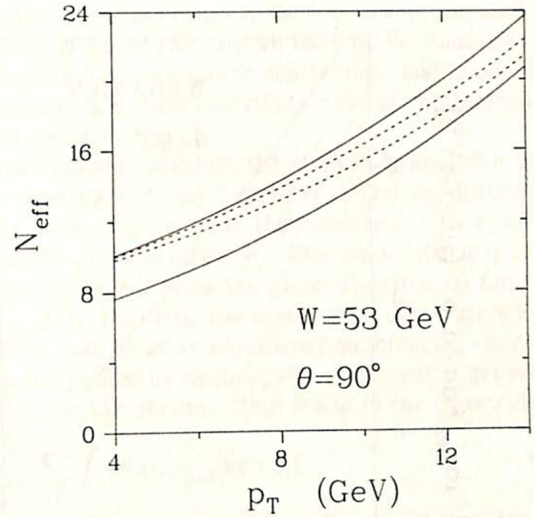

FIG. 4. Effective power at constant s, N_{eff}, versus p_T for pion production at $\sqrt{s} = 53$ GeV, $\theta = 90°$. The upper and lower solid curves are the predictions of MQSM and OGE, respectively. Both models are calculated using scale-violating quark distribution functions, Eq. (3.4). The dashed curves are the result of adding MQSM and OGE together and then extracting N_{eff}. The lower dashed curve corresponds to the higher range of $\alpha_s(-\hat{t})$.

for N_{eff} when MQSM and OGE are added together. [The upper dashed curve corresponds here to the smaller choice for $\alpha_s(-\hat{t})$.] In the region $p_T \gtrsim 10$ GeV, where the OGE contribution might be expected to dominate, we see that $N_{\text{eff}}^{\text{OGE}} > 12$.

We think that a better approach would be to take advantage of the fact that these models predict the normalization of the inclusive cross section as well as its p_T dependence. At given values of p_T and $\sqrt{s}$ it is easy to calculate from (1.2), (1.3), and (3.4) what the inclusive cross section would be for any value of α_s if OGE dominates. A reasonable approach would then be to calculate α_s^{eff} from the data and a given set of assumptions about the structure functions. This would then correspond to $\alpha_s(\langle -\hat{t}\rangle)$, where $\langle \hat{t}\rangle$ is the average momentum transfer in the process. At high $\langle -\hat{t}\rangle$ this should approximately correspond to $\alpha_s(Q^2)$ determined from other calculations. Given that the Field-Feynman MQSM parametrization of $d\sigma/d\hat{t}$ is a good fit to the ISR data, $\alpha_s(\langle -\hat{t}\rangle)$ determined in this manner from present data would be close to the curve labeled FF in Fig. 1. At higher values of $\langle -\hat{t}\rangle$ it should flatten out and be consistent with the QCD expectations of a logarithmic dependence. When this happens, it would also be interesting to check that the angular dependence of the cross section is consistent with OGE.

One might also hope to observe OGE in the production of hadronic jets. In Fig. 5 we show the

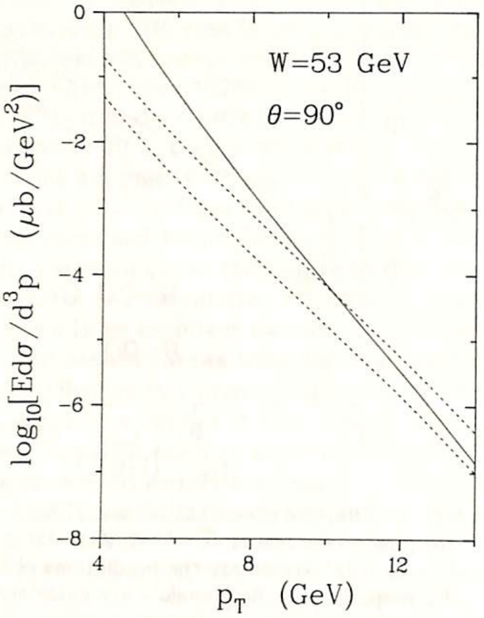

FIG. 5. Inclusive cross section for jet production, $E\,d\sigma/d^3p\,(pp\to \text{jet}+X)$, versus p_T at $\sqrt{s}=53$ GeV, $\theta=90°$. The solid curve is calculated using the Field and Feynman parametrization of $d\sigma/d\hat{t}$. The dashed curves are the results of one-gluon exchange corresponding to the range of $\alpha_s(-\hat{t})$ shown in Fig. 1 as dashed curves. Both models are calculated using scale-violating quark distribution functions, Eq. (3.4).

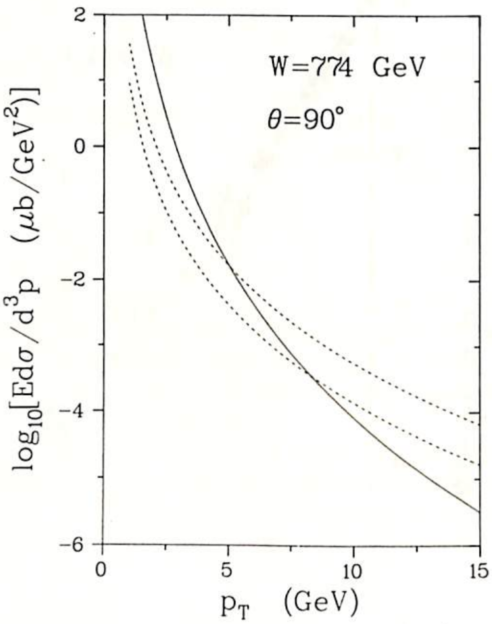

FIG. 6. Inclusive cross section for pion production, $E\,d\sigma/d^3p\,(pp\to \pi X)$, versus p_T at $\sqrt{s}=774$ GeV, $\theta=90°$. The solid curve is calculated using the Field and Feynman parametrization of $d\sigma/d\hat{t}$. The dashed curves are the results of one-gluon exchange corresponding to the range of $\alpha_s(-\hat{t})$ shown in Fig. 1 as dashed curves. Both models are calculated using scale-violating quark distribution functions, Eq. (3.4).

inclusive cross section for jet production at $\sqrt{s}=53$ GeV obtained by setting $D_{\pi/c}(z)=\delta(1-z)$ in (1.2). Comparing jet production (Fig. 5) to π^0 production (Fig. 3), one sees that the jet cross sections are about 100 times larger than the π^0, but that the crossover between OGE and FF has moved to slightly higher values of p_T.

C. Large-p_T hadron experiments at new accelerators

One of the first experiments at a new proton-proton intersecting-storage-ring facility[34] should be to measure the large-p_T inclusive hadron yield. We would like to see how sensitive these measurements might be to the presence of a fundamental one-gluon-exchange quark-quark interaction.

Given our assumptions, we can calculate with (1.2) using the Field and Feynman model (MQSM) and OGE with α_s given by (2.22). We assume that MQSM gives a good extrapolation to higher energies of the current trend in the data. (Our conclusions would not change substantially if we were to use the CIM instead of MQSM). In Fig. 6 we show the predictions of these models for π^0 production at $\sqrt{s}=774$ GeV (150×1000-GeV colliding rings), using the scale-violating structure functions (3.4). Compared to the results at $\sqrt{s}=53$ GeV, the crossover between MQSM and OGE now occurs at lower values of p_T, and both cross sections have increased. The two models are clearly distinguishable for $p_T\gtrsim 8$ GeV. In Fig. 7 we show N_{eff} at $\sqrt{s}=774$ GeV, and again the models are easily distinguished. At these energies N_{eff} is close to the values expected from scaling arguments. Figure 8 shows the cross sections for jet production at this energy. The crossover between OGE and FF occurs at significantly higher values of p_T ($p_T=8$–13 GeV) for jet production than for π^0 production ($p_T=6$–8 GeV).

The proposed new colliding-ring facilities should give us a clear answer concerning the presence of the QCD term if the luminosities are high enough to measure inclusive pion yields on the order of 10^{-4} μb/GeV^2.

D. Hard scattering from gluons

We have been careful to interpret our calculations as lower limits for the inclusive single-particle data. This is because we only calculate one contribution to (1.2), and there are many other internal processes which could contribute. In the hard-scattering limit, where interference effects are presumed negligible, these processes will add

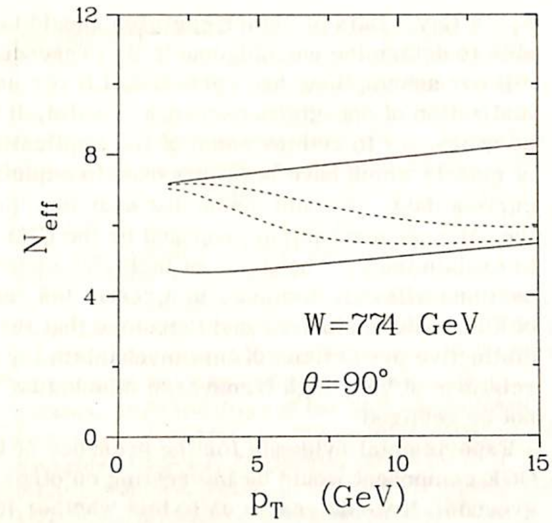

FIG. 7. Effective power at constant s, N_{eff}, versus p_T for pion production at $\sqrt{s} = 774$ GeV, $\theta = 90°$. The upper and lower solid curves are the predictions of MQSM and OGE, respectively. Both models are calculated using scale-violating quark distribution functions, Eq. (3.4). The dashed curves are the results of adding MQSM and OGE together and then extracting N_{eff}. The lower dashed curve corresponds to the higher range of $\alpha_s(-\hat{t})$.

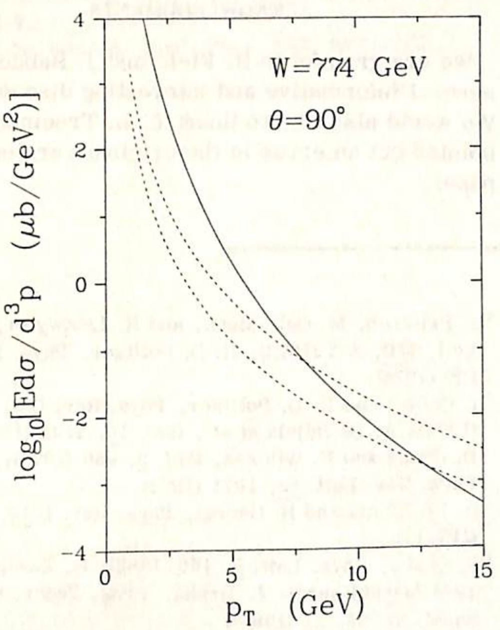

FIG. 8. Inclusive cross section for jet production, $E d\sigma/d^3p\,(pp \to \text{jet}+X)$, versus p_T at $\sqrt{s} = 774$ GeV, $\theta = 90°$. The solid curve is calculated using the Field and Feynman parametrization of $d\sigma/d\hat{t}$. The dashed curves are the results of one-gluon exchange corresponding to the range of $\alpha_s(-\hat{t})$ shown in Fig. 1 as dashed curves. Both models are calculated using scale-violating quark distribution functions, Eq. (3.4).

to the cross section. In QCD quark-gluon and gluon-gluon scattering we have p_T dependence similar to quark-quark scattering, and hence these processes should contribute even at the largest values of p_T.

We cannot calculate the contributions from these processes with any degree of certainty, although we know they occur to the same order in α_s as in quark-quark scattering. The main difficulty is that we do not know the gluon distribution function, $G_{V/p}(x)$. By using the momentum sum rule we can infer that there is significant momentum carried in the proton by uncharged constituents, presumably vector gluons. This leads to the constraint

$$\sum_{i,j} \int dx\, x G_{V^i_j/p}(x) \cong 0.5\,, \tag{3.8}$$

where $i, j = 1-3$ are color indices, but this does not help us determine the shape of the gluon distribution. If we make theoretical assumptions such as constituent-counting rules,[35] or if we apply a bremsstrahlung model for the emission of gluons from valence quarks,[36] then we can do slightly better and deduce some of the qualitative features of the gluon distribution. These approaches indicate that the gluons are concentrated at smaller values of x than the valence quarks. At large p_T and $x_T = 2p_T/s^{1/2} \neq 0$ it may therefore be reasonable to neglect the contribution to (1.2) from quark-gluon and gluon-gluon scattering compared to $qq \to qq$. As $s \to \infty$ and $x_T \to 0$ we should begin to see some effects.

Although we will not address this question further here, the contribution of gluons to large-p_T hadronic collisions is a topic worth pursuing. Aside from the intriguing suggestion of Einhorn and Ellis[37] that gluon-gluon annihilation could describe production of even-C-parity charmonium states (η_c, χ) in pp collisions, there have been few instances in the literature where hard-scattering ideas have been applied to the vector-gluon content of hadrons.

Another matter which we have ignored is the effect of the transverse-momentum distribution of the quarks within the initial hadrons. The simplest guess is that this effect will shift all our calculated OGE curves to the right by about 0.3 GeV. At any rate, the effect can only increase the calculated inclusive cross sections above what we present as a lower bound.

IV. SUMMARY AND CONCLUSIONS

In many ways, the study of large-p_T hadronic collisions has been the neglected stepchild of the quark-parton model. Many theoretical techniques by which QCD has been applied to e^+e^- annihilation or deep-inelastic lepton scattering have no direct application here. Although we have no rigorous framework based on asymptotically free field theories to support quark-parton ideas here, nevertheless we do have accumulating experimental evidence that 2-2 hard-scattering models are valid.[38] The experimental support therefore enables us to proceed cautiously, relying heavily on intuition and analogy with leptonic processes. There is one element, at least, of this analogy which seems moderately straightforward and which should merit some degree of confidence. That element involves the existence of an approximately scale-invariant component to the inclusive cross sections arising from the pointlike scattering of two quarks. At various times there have been proposals that this component can be suppressed or absent,[39] but none of these arguments seem particularly compelling and we prefer to proceed on the assumption that the process is present.

If we assume that the OGE piece of qq scattering does exist, we can use our analogy with lepton processes to normalize it. Two main assumptions are necessary. We assume that the effective coupling in QCD between a quark and a gluon with large spacelike momentum is approximately the same in qq elastic scattering as it is when the quarks are far off the mass shell. This effective coupling can then be determined elsewhere. We also assume that the scaling violations in the quark distribution functions are similar to those seen in deep-inelastic lepton processes. With these assumptions, our calculations show that the OGE contribution to the cross section may be a significant contributor to $pp \to \pi X$ at $\sqrt{s} = 53$ GeV and $p_T \geq 9$ GeV. Data at higher energies should be able to determine unambiguously its presence.

If our assumptions are correct and if our normalization of one-gluon-exchange is valid, it will be necessary to rethink some of the implications of models which have been proposed to explain current data. It would mean, for example, that the $qM \to qM$ mechanism proposed by the CIM to explain the p_T^{-8} behavior of inclusive cross sections will only dominate in a restricted range of kinematic variables, and therefore that the distinctive predictions of this mechanism for correlations at very high transverse momentum[40] may not be realized.

Experimental evidence for the presence of the OGE component would be interesting on other grounds. It would enable us to test whether ideas extracted from QCD and asymptotic freedom are applicable to purely hadronic processes. At a time when proton machines are being proposed as "quark factories," it would be comforting to know that the interactions of hadrons can be well described by the interactions of their constituents. These calculations also enable us to estimate hadron yields at very high p_T where they might provide background for interesting W or Z production.

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